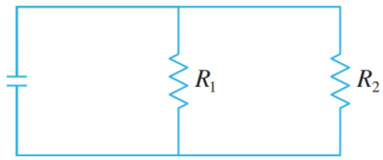
Exercise 39

If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of $0.3\ \Omega/\text{s}$ and $0.2\ \Omega/\text{s}$, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



Solution

Simplify the right side of the given formula.

$$\begin{aligned}\frac{1}{R} &= \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2} \\ &= \frac{R_2 + R_1}{R_1R_2}\end{aligned}$$

Invert both sides to get R.

$$R = \frac{R_1R_2}{R_2 + R_1}$$

Take the derivative of both sides with respect to time by using the chain and quotient and product rules.

$$\begin{aligned}\frac{d}{dt}(R) &= \frac{d}{dt}\left(\frac{R_1R_2}{R_2 + R_1}\right) \\ \frac{dR}{dt} &= \frac{\left[\frac{d}{dt}(R_1R_2)\right](R_2 + R_1) - \left[\frac{d}{dt}(R_2 + R_1)\right]R_1R_2}{(R_2 + R_1)^2} \\ &= \frac{\left(\frac{dR_1}{dt}R_2 + R_1\frac{dR_2}{dt}\right)(R_2 + R_1) - \left(\frac{dR_2}{dt} + \frac{dR_1}{dt}\right)R_1R_2}{(R_2 + R_1)^2}\end{aligned}$$

Therefore, when $dR_1/dt = 0.3$, $dR_2/dt = 0.2$, $R_1 = 80$, and $R_2 = 100$, the rate of change of the total resistance is

$$\left.\frac{dR}{dt}\right|_{\substack{R_1=80 \\ R_2=100}} = \frac{[(0.3)(100) + (80)(0.2)](100 + 80) - (0.2 + 0.3)(80)(100)}{(100 + 80)^2} = \frac{107}{810}\ \frac{\Omega}{\text{s}} \approx 0.132099\ \frac{\Omega}{\text{s}}.$$